

Power-law behavior observed in p_T -distributions and its implications in relativistic heavy-ion collisions

MENG Ta-chung *

Department of Physics, CCNU, 430079 Wuhan, China and

Institut für Theoretische Physik, FU-Berlin, 14195 Berlin, Germany

LIU Qin †

Department of Physics, CCNU, 430079 Wuhan, China

(Dated: February 9, 2008)

Preconception-free analyses of the inclusive invariant transverse-momentum distribution data taken from the measurements of Au+Au collisions at $\sqrt{s_{NN}} = 130$ GeV and $\sqrt{s_{NN}} = 200$ GeV have been performed. It is observed that the distributions exhibit for $p_T \geq 2$ GeV/c remarkably good power-law behavior (p_T -scaling) with general regularities. This power-law behavior leads us in particular to recognize that the concept of centrality, albeit its simple appearance, is rather complex; its underlying geometrical structure has to be understood in terms of fractal dimensions. Experimental evidences and theoretical arguments are given which show that the observed striking features are mainly due to geometry and self-organized criticality. A simple model is proposed which approximately reproduces the above-mentioned data for the “suppression” without any adjustable parameter. Further heavy-ion collision experiments are suggested.

PACS numbers: 25.75.-q, 25.75.Dw, 05.65.+b, 13.85.Hd, 13.87.Ce

Inclusive invariant p_T -distributions for charged hadrons in Au+Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV have been measured and published by STAR [1, 2] and by PHENIX [3] over a broad range of centrality. Such p_T -distributions for neutral pions are also given by PHENIX [4]. All these experiments [1, 2, 3, 4] show that the hadron yields differ appreciably at high and medium p_T in central collisions relative to peripheral collisions and to the nucleon-

* Email address: meng@mail.ccnu.edu.cn; meng@physik.fu-berlin.de

† Email address: liuq@mail.ccnu.edu.cn

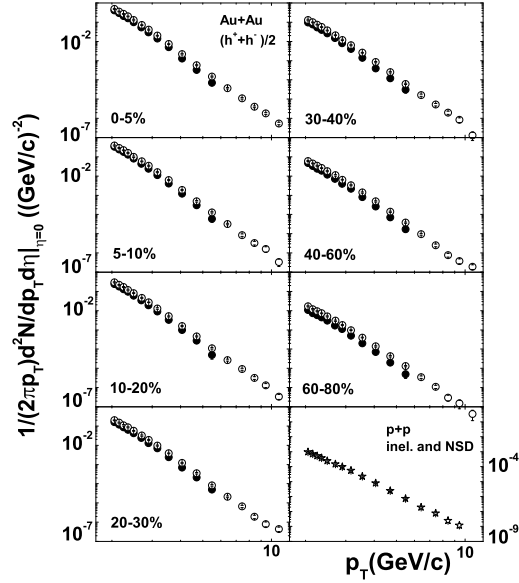


FIG. 1: Inclusive invariant p_T -distribution data [1, 2] for $p_T \geq 2$ GeV/c in log-log plots. Black and white indicate 130 and 200 GeV respectively.

nucleon reference. What do these observations, usually known as “suppression” [5], tell us? Are they related to the yet-to-be-found “quark-gluon-plasma (QGP)”? If yes, how? In order to obtain an unbiased physical picture to start with, we begin with preconception-free data-analyses. We then summarize the results, and discuss their implications.

In *the first part* of this paper, we report on the result of such analyses. Within the measured kinematical region $0 < p_T < 12$ GeV/c, $\sqrt{s_{NN}} = 130$ GeV and $\sqrt{s_{NN}} = 200$ GeV, the inclusive invariant p_T -distributions of $(h^+ + h^-)/2$ for centrality-selected Au+Au, and those for p+p interactions exhibit power-law behavior for $p_T \geq 2$ GeV/c (cf. Fig. 1). The results can be summarized as follows.

$$\frac{1}{2\pi p_T} \frac{d^2 N}{dp_T d\eta} \Big|_{\eta=0} (p_T | Au + Au; p_c) \propto p_T^{-\lambda_{AuAu}(p_c)}, \quad (1)$$

$$\frac{1}{2\pi p_T} \frac{d^2 N}{dp_T d\eta} \Big|_{\eta=0} (p_T | p + p) \propto p_T^{-\lambda_{pp}}, \quad (2)$$

where the power-indices (the λ 's) are positive real numbers, and p_c characterizes the centrality-bins (in percentile) which stand for the different degrees of departure from the most central collision. The experimental values of the λ 's obtained from the STAR data

[1, 2] at $\sqrt{s_{NN}} = 130$ GeV and 200 GeV for different p_c -bins are shown in Fig. 2. The results from the PHENIX data [3, 4] (will be reported in a more extensive paper elsewhere [6]) show similar characteristic properties.

As we can see in Fig. 2, for a given p_c -bin, the λ -value at $\sqrt{s_{NN}} = 200$ GeV coincides with the corresponding one at $\sqrt{s_{NN}} = 130$ GeV. Furthermore, the λ -value for the most peripheral ($p_c \rightarrow 100\%$) case is very much the same as λ_{pp} 's. Note that the λ -values increase from the most peripheral value, $\lambda_{AuAu}(p_c \rightarrow 100\%) \approx \lambda_{pp}$, with decreasing p_c to the λ -value for the most central (center-on-center) collision ($p_c \rightarrow 0\%$) in a monotonous manner. In this connection it is useful to consider the ratio of both sides of Eqs. (1) and (2):

$$\frac{d^2N/p_T dp_T d\eta|_{\eta=0}(p_T|Au + Au; p_c)}{d^2N/p_T dp_T d\eta|_{\eta=0}(p_T|p + p; inel.or NSD)} \propto p_T^{-\lambda(p_c)}. \quad (3)$$

The quotient, which is a completely experimental quantity, on the left-hand-side of this equation, will hereafter be referred to as $Q(p_T, p_c)$; and the values of the exponent on the right-hand-side,

$$\lambda(p_c) = \lambda_{AuAu}(p_c) - \lambda_{pp}, \quad (4)$$

are depicted in Fig. 3. They are directly obtained from the data points shown in Fig. 2.

In *the second part* of this paper, we propose a simple model. We show how the power-law behavior can be understood, and how $\lambda_{AuAu}(p_c)$ and λ_{pp} in Eqs. (1) and (2) can be estimated. The model is based on geometry and self-organized criticality (SOC) [7, 8]. As we shall see, both geometry and SOC contribute powers in p_T to the distributions shown in Eqs. (1) and (2). The relevant facts and arguments are given below:

(A) Geometry: Let us first recall the well-known observation made by Rutherford [9] on large-momentum-transfer scattering, and a less-known observation made by Williams [10] in which the following has been pointed out: Ordinary space-time concepts are useful for the semiclassical description of high-energy scattering processes, *provided that* the de Broglie wavelength of the projectile is short compared to the linear dimension of the scattering field, *and provided that* the corresponding momentum transfer which determines the deflection is not smaller than the disturbance allowed by the uncertainty principle. Through a simple realistic estimation, we can, and we did, convince ourselves that all these conditions *are indeed fulfilled* for Au+Au and p+p collisions at $\sqrt{s_{NN}} \geq 130$ GeV and $p_T \geq 2$ GeV/c. Furthermore we note that the corresponding phase-space factors can be estimated by making use of the uncertainty principle in accordance with Refs. [9] and [10].

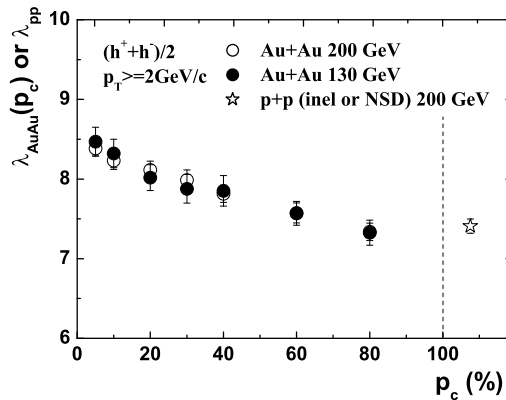


FIG. 2: The power-indices, $\lambda_{AuAu}(p_c)$ and λ_{pp} , evaluated by measuring the slopes in Fig. 1 are plotted as function of p_c .

(B) SOC: It is well-known that approximately 50% of the kinetic energy of every fast moving hadron is carried by its gluonic content and that the characteristic properties of the gluons, in particular, the direct gluon-gluon coupling prescribed by the QCD Lagrangian, the confinement, and the nonconservation of gluon numbers, can and should be considered as more than a hint that systems of interacting soft gluons are open dynamical complex systems which are far from thermal and/or chemical equilibrium. Taken together with the observations [7, 8] made by Bak, Tang, and Wiesenfeld (BTW), these facts strongly suggest the existence of SOC and thus the existence of BTW avalanches in gluonic systems [11, 12].

According to SOC, a small part of such BTW avalanches manifests themselves in the form of color-singlet gluon clusters c_0^* , and that they can be readily examined [11, 12] experimentally in inelastic diffractive scattering processes [13]. This is because the interactions between the struck c_0^* and any other color singlets are of Van der Waal's type which are much weaker than color forces at distances of the order of hadron radius. In fact, in order to check the existence and the properties of the c_0^* 's, a systematic data analysis has been performed [12], the result of which shows that the size distribution $D_S(S)$, and the lifetime distribution $D_T(T)$ of such c_0^* 's indeed exhibit power-law behavior $D_S(S) \propto S^{-\mu}$, $D_T(T) \propto T^{-\nu}$, where μ and ν are positive real constants. Such characteristic features are known as “the fingerprints of SOC” [7, 8]. These fingerprints imply in inelastic diffractive scattering, the size S of the struck c_0^* is proportional to the directly measurable quantity x_P , which is the energy fraction carried by “the exchanged colorless object” in such processes, the existing data [13] show

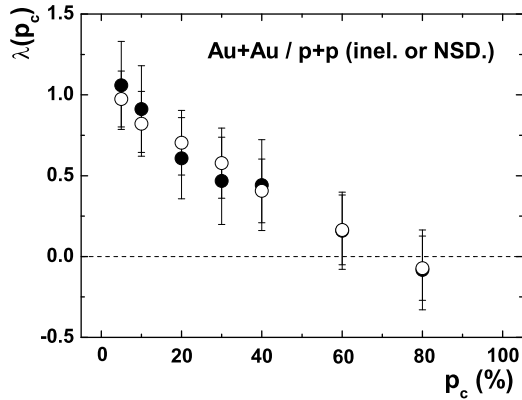


FIG. 3: The power-index $\lambda(p_c)$ defined in Eq. (4) plotted as function of p_c . Data are from Refs. [1, 2].

$D_S(x_P) \propto x_P^{-\mu}$, where $\mu = 1.95 \pm 0.12$ [11, 12].

By considering inelastic diffractive scattering [11, 12], we were able to check—and only able to check—the existence and properties of the color-singlet gluon clusters. But, due to the observed SU(3) color symmetry, most of such gluon clusters are expected to be color multiplets which will hereafter be denoted by c^* 's. Furthermore, in accordance with the experimentally confirmed characteristic features of the BTW theory, the SOC fingerprints in gluon systems should not depend on the dynamical details of their interactions, in particular, not on the details about the exchanged quantum numbers in their formation processes. This implies that $D_S(S)$ and $D_T(T)$ of the c^* 's are expected to have [14] not only the same power-law behavior but also the same power as that of their color-singlet counterparts observed in inelastic diffractive scattering processes [11, 12].

The fact [13] that quarks can be knocked out of protons by projectiles whenever large-momentum-transfer between projectiles and targets take place, has prompt us to propose [14] that c^* 's can also be “knocked out” of the mother proton by a projectile provided that the corresponding transfer of momenta is large enough where the knocked-out c^* 's may have “color-lines” connected to the remnant of the proton. What we show now is that the observed power-law behavior in Eq. (2) can be quantitatively described by the product of the probability distribution(s) of the knocked-out c^* 's and the phase-space factors associated with the knock-out processes.

Recall that processes of inclusive high- p_T jet-production, $p+p \rightarrow \text{jet} + X$, at high energies

are dominated by two-jet events; and that in a SOC-based model [14], such jets are produced in two-step-processes. In Step 1: A quark q (q_v or q_s or \bar{q}_s) in one of the colliding nucleon collides with a quark q (q_v or q_s or \bar{q}_s) in the other nucleon where an amount of p_T is transferred in the plane in which the two nucleons in form of thin contracted objects meet each other, and in which large- p_T quark-quark scattering takes place. In Step 2: Since the two scattered q 's and/or \bar{q} 's are in general space-like (because of the large p_T), the easiest way for them to escape the confinement region is each “catches” a suitable time-like (in order to provide the high- p_T q or \bar{q} with sufficient energy) anticolor BTW-avalanches, c^* 's, which in accordance with the SOC-picture exist in abundance in their neighborhood. This is how a color-singlet jet is created. A scattered q or \bar{q} can also combine with a colored c^* to form a jet or a fan which is connected with other colored object(s) through color-lines. This is how color multiplet jets (or fans) can be produced. Hence, in the proposed picture the invariant cross section $Ed^3\sigma/dp^3$ is expected to have the following factors.

(i) A phase-space factor that describes the chance for the two quarks (q or \bar{q} , \dots) which initiate Step 1 to come so close to each other in space that they can exchange a large p_T ($\approx E_T$). This phase-space factor can be estimated by making use of the uncertainty principle and the two observations mentioned in (A) above. By choosing the z axis as the collision axis, p_T will be in (or near) the xy plane. The chance for two constituents (q or \bar{q} , \dots) moving parallel to the z axis to come so close to each other in the xy plane such that an amount p_T can be transferred is approximately proportional to the size of the corresponding phase space $\Delta x \Delta y \sim (\Delta p_x)^{-1} (\Delta p_y)^{-1} \sim p_T^{-1} p_T^{-1} = p_T^{-2}$.

(ii) Since each jet is associated with a to be knocked-out gluon cluster c^* , which has energy comparable with E_T , $Ed^3\sigma/dp^3$ is expected to be proportional to the square of the probability D_S to find such a c^* . The size S of BTW avalanches is directly proportional to x_p , thus proportional to the energy E_T it carries. Furthermore, since $E_T \approx p_T$ for high-energy jets [14], we have:

$$D_S(x_p) \propto D_S(E_T) \propto p_T^{-2\mu}. \quad (5)$$

This means, we expect to see a factor $p_T^{-2\mu}$ in the invariant cross section $Ed^3\sigma/dp^3$.

(iii) Having in mind that the scattered quarks are in general space-like, two-step-processes are expected to take place only when there are suitable c^* 's in the surroundings immediately after the first step. The probability of having sufficient c^* 's around, wherever and whenever two constituents meet during the p+p collision, is guaranteed when the c.m.s of one proton

meets that of the other. Hence, phase-space considerations w.r.t time requires a factor $(\Delta t)^2 \sim E_T^{-2} \sim p_T^{-2}$.

Hence, for p+p collisions, we expect to see $Ed^3N/dp^3 \propto Ed^3\sigma/d^3p \propto p_T^{-2-2-2\mu}$. By taking the lower limit of μ , we obtain:

$$\frac{1}{2\pi p_T} \frac{d^2N}{dp_T d\eta} \Big|_{\eta=0}(p_T|p+p) \propto p_T^{-7.66} \quad (6)$$

which is in reasonable agreement with the data [2].

Next, we focus our attention to the *empirical* result described by Eqs. (3) and (4) together with Fig. 3. Note that according to Eq. (3), the quotient $Q(p_T, p_c)$ stands for the chance to find a large- p_T charged hadron in Au+Au collisions within a given p_c range; and this chance is measured in “units” of the chance in finding a similar large- p_T hadron in p+p collisions. We now take a closer look at the straight lines, on which the data-points lie in the log-log plots of Fig. 1, and note the fact that the slopes depend only on p_c —independent of $\sqrt{s_{NN}}$. This has to be considered as a strong hint at the possible distinguished role played by geometry in describing/understanding such collision processes. Hence, it is useful not only to recall the facts and the arguments mentioned in (A) and those in (i) above, but also to recall that the word “centrality” p_c is in fact very much involved: Experimentally [1, 3] it is determined by measuring the multiplicities of produced hadrons; but, since the notion of “*departure from the center-on-center case*” is geometrical, it is expected to be describable in terms of geometry, in particular in terms of impact parameters, b ’s. These facts and arguments have led us to propose the following picture. High- p_T jet-production processes in relativistic heavy-ion (AA) collisions can be viewed as an ensemble of corresponding jet-production processes in binary nucleon-nucleon (NN) collisions. The observed effects depend significantly on collision-geometry.

Since every AA event corresponds to a b -parameter (note that the reverse is not true), an ensemble of collision events corresponds to a set of b -parameters. By choosing the z -axis as the collision axis where the centers of the two colliding nuclei are located at $(-b/2, 0)$ and $(b/2, 0)$, on the x -axis of the xy -plane in every event, we obtain point sets of impact parameters. It is on such point sets, *the geometrical support*, we study the above-mentioned p_T -distributions. Having the observations made by Rutherford [9] and Williams [10] in mind [cf. (A) and (i) above], the relation $\Delta x \sim (\Delta p_x)^{-1} \sim p_T^{-1}$ obtained by using the uncertainty principle tells us the following. For every measured value p_T , there is an interval Δx ; and it

is with *this precision* in the corresponding spatial coordinate that the probability $Q(p_T, p_c)$ [precisely speaking $Q(\Delta x, p_c)$ on its geometrical support] of finding high- p_T charged hadrons can be measured.

In the proposed picture based on SOC and geometry, we are not (at least not yet) in a position to make predictions for $Q(\Delta x, p_c)$ or its geometrical support—not even the dimensions of such object! But fortunately, we know how to measure them! Thank the master-mathematicians: K. Weierstrass, G. Cantor, H. von Koch, F. Hausdorff, \dots , P. Lévy and B. Mandelbrot [15], we learned how to use the box-counting technique. Due to the facts and the arguments given in (A) and (i) above, we know that the length of the boxes in our case are of the order $\Delta x \sim (\Delta p_x)^{-1} \sim p_T^{-1}$ which implies: Δx becomes smaller and smaller for larger and larger p_T . Hence the observed p_T -scaling tells us that the result of this box-counting is nothing else but the result summarized in Eq. (3) which can also be written as $Q(\Delta x, p_c) \propto (\Delta x)^{\lambda(p_c)}$. Since this observation is independent of the positions of the boxes in each p_c -bin, by normalizing the probabilities $Q(\Delta x, p_c)$, the number of boxes, $N(\Delta x, p_c)$, needed to cover the produced hadrons distributed on the geometrical support is proportional to the inverse of $Q(\Delta x, p_c)$. That is: $N(\Delta x, p_c) \propto (\Delta x)^{-\lambda(p_c)}$. Hence, in the limit of large p_T , thus small Δx , $\lambda(p_c)$ is the corresponding fractal dimension of the geometrical support which consists of set of impact parameter points within each given p_c -bin. This means, in the proposed model, we expect to see that the inclusive invariant p_T -distributions for $p_T \geq 2$ GeV/c in any kind of relativistic heavy-ion (AA) collisions satisfy

$$\frac{1}{2\pi p_T} \frac{d^2 N}{dp_T d\eta} \Big|_{\eta=0}(p_T | AA; p_c) \propto p_T^{-\lambda_{NN} - \lambda(p_c)} \quad (7)$$

where $\lambda_{NN} \approx 7.66$, and the $\lambda(p_c)$'s are the fractal dimensions of the point-set of impact parameters.

It would be exciting to see further experiments with larger p_T -values, at higher energies and with other kinds of colliding nuclei. Such experiments will not only serve the general goal of QGP-search, but also, in particular, be able to check whether the empirical regularities are indeed as general as the existing data seem to suggest, and to check whether/how concepts and methods of Complex Sciences in particular those borrowed from Fractal Geometry and SOC are helpful in understanding Relativistic Heavy-Ion Physics.

The authors thank XU Nu for explaining the experiments of STAR Collaboration to us. We thank LIANG Zuo-tang and XU Nu for their critical remarks and useful suggestions.

We also thank LAN Xun and LIU Lei for helpful discussions and KeYanChu of CCNU for financial support. This work is supported by the National Natural Science Foundation of China under Grant No. 70271064.

-
- [1] C. Adler *et al.*, Phys. Rev. Lett. **89**, 202301 (2002).
 - [2] J. Adams *et al.*, Phys. Rev. Lett. **91**, 172302 (2003).
 - [3] K. Adcox *et al.*, Phys. Rev. Lett. **88**, 022301 (2002).
 - [4] S.S. Adler *et al.*, Phys. Rev. Lett. **91**, 072301 (2003).
 - [5] The precise definition of “suppression” can, e.g., be found in Refs [1, 2, 3, 4] and the references herein.
 - [6] Q. Liu, L. Liu, X. Lan and T. Meng (in preparation).
 - [7] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987); P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. A **38**, 364 (1988).
 - [8] P. Bak, *How Nature Works* (Springer, New York, 1996); H. J. Jensen, *Self-Organized Criticality* (Cambridge University Press, Cambridge, UK, 1998).
 - [9] E. Rutherford, Philos. Mag. **XXI**, 669 (1911).
 - [10] E. J. Williams, Rev. Mod. Phys. **17**, 217 (1945).
 - [11] T. Meng, R. Rittel, and Y. Zhang, Phys. Rev. Lett. **82**, 2044 (1999).
 - [12] C. Boros, T. Meng, R. Rittel, K. Tabelow, and Y. Zhang, Phys. Rev. D **61**, 094010 (2000).
 - [13] See, e.g., H. Abramowicz and A. Caldwell, Rev. Mod. Phys. **71**, 1275 (1999); A. M. Cooper-Sarkar, R.C. E. Devenish, and A. de Roeck, Int. J. Mod. Phys. A **13**, 3385 (1998), and references therein.
 - [14] J. Fu, T. Meng, R. Rittel and K. Tabelow, Phys. Rev. Lett. **86**, 1961 (2001).
 - [15] B.B. Mandelbrot, Science **155**, 636 (1967). B.B. Mandelbrot, *The Fractal Geometry of Nature*, W.H. Freeman, N.Y. G.A. Edgar, *Classics on fractals*, Westoiew, 2004.